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When the variables have a common factor >1, the representation is called improper.

Theorem. *There exists no representations as the sum of two squares for a number e having a prime factor of the form* $4m+3$ *occurring to an odd power; no proper representations when such a factor occurs to an even power. If* $P=p_1^{\pi_1}\dots p_s^{\pi_s}$, *where* $p_1, \dots, p_s$ *denote all the distinct primes of the form* $4m+3$ *which divide e, and if* $\pi_1, \dots, \pi_s$ *are all even, there are as many improper representations of e as there are representations of e/P; every representation of e is of the type* $(P^{\frac{1}{2}} x)^2+(P^{\frac{1}{2}} y)^2$.

Examples. $N_e=0$ for $e=3, 7, 15, 21, 27, 63$; $N_9=1$, $N_{45}=2$, $N_{225}=3$, $45=3^2+6^2=6^2+3^2$, $9.25=15^2=9^2+12^2=12^2+9^2$.

The problem thus reduces to the case in which every prime factor of e is of the form $4m+1$. Then by (2),

$$N_e=(\pi_1+1)(\pi_2+1)\dots(\pi_n+1). \tag{3}$$

For example, $N_5=N_{10}=2$, $N_{25}=3$, $25=5^2=3^2+4^2=4^2+3^2$.

When e is of the form $2^a p$, we have $N_e=2$. Removing the restriction that j^2+k^2 and k^2+j^2 shall be regarded as distinct representations, we obtain the well-known theorem of Fermat:

Every prime number p of the form $4m+1$ (*and every product* $2^a p$) *can be represented in one and only one way as the sum of two squares.*

In general, the representations enumerated in (3) include improper ones. For instance, if $\pi_1>1$, the $(\pi_1+1)(\pi_2+1)\dots$ representations x^2+y^2 of e/p_1^2 yield improper representations $(p_1 x)^2+(p_1 y)^2$ of e. There are only 2^{n-1} distinct representations of e not distinguishing the order or signs of the variables in x^2+y^2 (Dirichlet-Dedekind, *Zahlentheorie*, p. 164).

A METHOD FOR CHANGING THE SCALE OF A NUMBER.

By C. E. WHITE, Nashville, Tennessee.

To convert a number in the scale s to a number in the scale $s-a$, where a may be positive or negative, the following process may be used.

Let $N=p_0p_1p_2p_3\dots p_n=p_n+sp_{n-1}+s^2p_{n-2}+\dots=f(s)$. By Taylor's theorem,

$$f(s)=f(a)+(s-a)f'(a)+\frac{(s-a)^2}{2\,!}f''(a)+\frac{(s-a)^3}{3\,!}f'''(a)+\dots$$

Dividing $f(a)$ by $(s-a)$ we get $q_0+\frac{r_0}{s-a}$ where q_0 is the quotient and r_0 the remainder, or $f(a)=r_0+q_0(s-a)$.

Hence, $f(s)=r_0+(s-a)[f'(a)+q_0]+\frac{(s-a)}{2!}f''(a)+\ldots$

In like manner, $f'(a)+q_0=r_1+q_1(s-a)$ and $\frac{f''(a)}{2!}+q_1=r_2+q_2(s-a)$, and so on. Then

$$f(s)=r_0+r_1(s-a)+r_2(s-a)^2+r_3(s-a)^3+\ldots+r_n(s-a)^n,$$
and $N=r_nr_{n-1}\ldots r_3r_2r_1r_0$, in the scale $s-a$.

Let it be required, for example, to convert 567834 in the scale of 12 to the scale of 11.

$$f(s)=4+3s+8s^2+7s^3+6s^4+5s^5.$$

The value of the functions, and of the q's and r's are shown below.

$f(1)$,	$f'(1)$,	$\frac{f''(1)}{2!}$,	$\frac{f'''(1)}{3!}$,	$\frac{f^{\text{iv}}(1)}{4!}$,	$\frac{f^{\text{v}}(1)}{5!}$
33	89 3	109 8	81 10	31 8	5 3
0	4	7	3	6	8

Hence, $N=863740$ in the scale of 11.

* * *

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

210. Proposed by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

Simplify, $\log[\sqrt[3]{(137)}\ \sqrt[7]{(56)}\div\sqrt[10]{(187)}\ \sqrt[4]{(75)}]$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and A. H. HOLMES, Brunswick, Me.

Let P be the value of the expression in the parentheses.
Then $\log P=\frac{1}{3}\log 137+\frac{1}{7}\log 56-\frac{1}{10}\log 187-\frac{1}{4}\log 75$.

A similar example is given in Dickson's *College Algebra*, p. 20, ex. 6. Professor Dickson gives an answer in accordance with the above solution. It is our opinion, and in this opinion concur Professor E. R. Hedrick of the Missouri State University, and Professor George Melcher of the Missouri State Normal School of Springfield, that the last sign should be +. There is quite a general agreement among mathematical writers, that the operations of multiplication and division should be performed in the exact order of their occurrence. Thus $4\times6\div3\times2=16$ and not 4. However, Professor Dickson says, were one to ask for the log of $ab\div cd$ one would surely give as an answer, $\log a+\log b-\log c-\log d$, and to this form the numerical form in the problem corresponds. While we recognize the force of Professor Dickson's argument, yet we believe that when there is a possibility of ambiguity, every doubt should be removed by an explicit notation or else by following a well established usage.